Modal and Buckling Analysis of a Cracked Cantilever Beam with Circular Cross Section

A Thesis Submitted in Partial Fulfillment of the Requirements for the degree of

B.Tech and M.Tech (Dual)

In

Civil Engineering

(Structural Engineering)

By

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710CE2018

Department of Civil Engineering

National Institute of Technology, Rourkela

Rourkela-769008

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UNDER THE GUIDANCE OF

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CERTIFICATE

This is to guarantee that the proposition entitled "**Modal and Buckling Analysis of a Cracked Cantilever Beam with Circular Cross Section**" put together by **S B SUBHAPRAKASH**, bearing Roll No **710CE2018** in the partial fulfillment of the necessities for the honor of **Master of Technology** (Dual degree) in Civil Engineering with specialization in Structural Engineering during 2014-15 season in National Institute of Technology, Rourkela is a legitimate work conveyed by him under my superintendence..

To, the best of my insight, the matter corporate in the theory has not been submitted to whatever other University/Institute for the recompense of any degree or certificate.

Place: NIT Rourkela | Prof. Uttam Kumar Mishra

Date: 1/6/2015 | Thesis Supervisor

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ABSTRACT

Engineering structures have structural defects such as cracks due to long term services, vibrations and material non-uniformity. Cracks in structural members lead to local changes in their stiffness and consequently their static and dynamic behavior is altered. Cracked beam problem has grabbed the attention of many researchers in recent years. Analytical, numerical and semi-numerical methods have been developed during the years.

The influence of cracks on free vibration and buckling load of the beam with a single transverse thorough surface crack using finite element method (FEM) is investigated in present work. In this study modal analysis of a cracked cantilever beam with circular cross section is analyzed for different crack positions and different crack depths using Ansys, Matlab and FFT Analyzer. Buckling analysis of the beam is also carried out using finite element method (FEM). Beam element of two degrees of freedom (transverse displacement and rotation) is taken for analysis. Stiffness matrix of intact beam element as well as the cracked beam element is computed from total flexibility matrix. Modal frequencies are verified using analytical and experimental procedures for different crack locations and different crack depths. Buckling load of the column is computed from the modal analysis.

CONTENTS

1	Introduction	1
2	Literature review	**5**
3	Theory and formulation	8
	3.1 Introduction	9
	3.2 Mathematical Formulation	9
4	Experimental analysis	15
	4.1 Introduction	16
	4.2 Specimen details	16
	4.3 Equipments required	16
	4.4 Set up and procedure for the test	19
5	Results and discussion	21
	5.1 Introduction	22
	5.2 Comparison with previous study	22
6	Conclusion	32
	References	34

List of symbols

The symbols are presented here for easy reference.

A	Cross-sectional area of the beam
a	Depth of crack
C_{ij}	coefficients of overall flexibility matrix
C_{intact}	flexibility matrix of intact beam element
C_{ovl}	overall additional flexibility matrix due to presence of crack
C_{total}	total flexibility matrix of cracked beam element
D	Diameter of the beam
E	young's modulus of elasticity
F_{I} and F_{II}	correction factors
G	strain energy release rate
h	depth of the beam
I	moment of inertia of the beam
J	polar moment of inertia of the section
K_{I}, K_{II} and K_{III}	stress intensity factors
K_e	Bending stiffness matrix of beam
L_1	Position of crack from the fixed end of the beam
L_e	Effective length of the beam
L_c	Distance between the crack position and free end of the beam
M_e	mass matrix of a cracked beam
P_{cr}	Critical buckling load
ρ	Density of the beam
$\xi\ and\ \xi'$	Penetrating depth of crack
ν	Poisson's ratio

LIST OF FIGURES

1	Uniform beam discretized into elements and free body diagrams of intact and cracked beam element with 2 degrees of freedom per node	10
2	A cracked beam element subjected to shear force and bending moment (Cross-sectional view)	11
3	Integration area A_S for calculating additional flexibility matrix of a beam with circular cross-section	12
4	A cracked beam element subjected to shear force and bending moment	14
5	Modal Impact Hammer (B&K type 2302-5)	19
6	Accelerometer (B&K 4507)	19
7	FFT Analyzer (B&K3560 B)	20
8	Display Unit	21
9	Vibration test setup, beam attached to the vice like cantilever beam, accelerometer and modal hammer in position	22
10	Graphical presentation of 1st fundamental modal frequencies versus depth of crack in Ansys 15	28
11	Graphical presentation of 1st fundamental modal frequencies versus depth of crack in FFT analyzer	28
12	Graphical presentation of 2nd fundamental modal frequencies versus depth of crack in FFT analyzer	29
13	Graphical presentation of 1st fundamental modal frequencies versus position of crack in Ansys 15	30
14	Graphical presentation of 1st fundamental modal frequencies versus position of crack in FFT analyzer	30
15	Graphical presentation of 2nd fundamental modal frequencies versus position of crack in Ansys 15	31

16	Graphical presentation of 2nd fundamental modal frequencies versus position of crack in FFT analyser	31
17	Graphical presentation of buckling load (Pcr) versus the depth of crack	32
18	Graphical presentation of buckling load (Pcr) versus position of crack	33

CHAPTER 1

INTRODUCTION

INTRODUCTION

Structures are proposed to bear up the loads they are required to be subjected during the service. Beams are the vital component of any designing applications. Beams which experience a wide range of static and dynamic loads give a substantial model to various structural applications. Airplane wings, helicopter rotor razor sharp edges, shuttle radio wires, and robot arms are structural sample can be displayed with beams.

Beams with variable cross-section and material properties are used as a part of aeronautical structures (e.g., rotor shafts and practically reviewed pillars), mechanical designing (e.g., robot arms and crane blasts), and structural designing.

It is also noteworthy that structures should securely work among its service period. Breakdown period is started if damage persists on the structures. Damage is termed as the sudden changes that brought into an arrangement, either by design or accidentally which impels a negative effect on the execution of that structure. Damage is a major stands in structural analysis because of safety measures and also financial advancement of the organizations. The presence of cracks is the most well-known structural defect. Local variations in stiffness are happened due to formation of cracks and structural defect in a member, the magnitude of which dependent on the location and depth of the crack. There are various reasons for the formation of cracks. Limited fatigue strength is one of the reason. For the most part the cracks are of little sizes and they are subjected to spread all through the beam. Owing to the absence of undetected cracks, there is a chance of sudden basic structural failure once they achieve their critical size. So natural frequency estimation is useful for the identification of crack. So early crack identification is imperative due to the fact that sudden structural failure because of heavy load operation instigates genuine harm or damage.

It is comprehended that the cracks in a structure yield an addition of the vibrational level, and diminish the load carrying capacity, and can constitute the purpose behind a danger. Vibration estimations give a non-damaging, beneficial and quick method to identify and locate cracks. Crack recognition is significant for structural health monitoring applications because of the fact that owing to static dynamic loadings cracks in a structure are risky. The SHM system incorporates the impression of a framework after sooner or later using discontinuously inspected dynamic reaction estimations from a mixture of sensors, the extraction of damage-delicate components from these estimations, and the factual examination of these elements to center the present state of framework wellbeing. Recently in two decades a considerable measure of exploration has been committed to add a powerful technique for methodology for crack identification. Early crack discovery is an essential part for guaranteeing security and unwavering quality of in-service structures. Due to mendacity and testability, the adjustments in modal frequencies of the structure previously, then after the fact damages are regularly used to demonstrate the condition of the structure. A considerable measure of exploration endeavors have been dedicated to adding to a compelling methodology for identification of crack in structures.

The structural security of beams is managed by few studies, notably crack identification by structural health monitoring. The change in natural frequencies and mode shapes of the beam is managed by the studies regarding structural health monitoring. Structural defects can be identified by vibrational analysis, for example, cracks of any structure offer a viable, reasonable and quick method for non-destructive testing. The stiffness and the damping in the structure is decreased due to the existence of crack or localized damage. It is stated by vibration theory that reduction in the stiffness of the structure is due to the decrease in the natural frequencies and change of the modes of vibration. Stiffness and damping properties of a structural element is changed by cracks or any defects and further it changes the dynamic behavior. Therefore, the natural frequencies and mode shapes of the structure contain where about of the damage. Local flexibilities are driven due to the cracks in the structure which affects the dynamic conduct of the entire structure to a notable degree. It results in reduction of natural frequencies and substantial changes in shape of the vibrations. Any analysis of these progressions makes it conceivable to distinguish cracks.

CHAPTER 2

LITERATURE REVIEW

LITERATURE REVIEW

Investigation of both coupling of longitudinal and bending vibrations of a rotating shaft due to surface crack was made by Papadopoulos et al. (1987).

An innovative method of analysis to observe the effect of two open cracks on the frequencies of the natural flexural vibrations in a cantilever beam was developed by ostachowicz et. al. (1991). In his study, both double-sided and single-sided cracks was considered.

Dimarogonas, (1996), states that the presence of crack in a structure affects its vibration response and local flexibility is induced. Moreover, the crack will open and close in time depending on rotation and vibration amplitude.

Chati et al., (1997), addresses the problems of a cracked beam due to vibrations. The motion of such beams are complex. The problem is due to the presence of non-linearity of opening and closing cracks.

A method to evaluate the lowest natural frequency for lateral vibration for beams with single edge breathing crack was suggested by Chondros et al., (1998).

Kisa et al., (1998), analyzed the vibrational characteristics of a cracked Timoshenko beam. FEM and component mode synthesis method is analyzed.

Calculation of natural frequencies for a beam with arbitrary numbers of transverse cracks was made by Shifrin et al. (1999) with a new approach. The dimension of the matrix, that involved in the calculation is decreased.Jagdale and Chakrabrati, analyzed the modal analysis of cracked beam for different boundary conditions with varying crack depths and crack positions.

Viola et al., (2001), stated that the dynamic behavior of the structure is changed due to local-flexibility introduced by the cracks and the cark position and the magnitude can be determined by analyzing this change. A special finite element model for a cracked Timoshenko beam is done for modelling in FEM analysis.

Zheng and Fan, (2001), presented a new method for the calculation of modal frequencies with a non-uniform beam taking random number of transverse open cracks. In this method a modified, Fourier series (MFS) specially developed for crack analysis was implemented.

Patil and Maiti, (2003), suggested a method for the detection of multiple open cracks in a slender euler-bernoulli beam is presented based on frequency measurements.

Zheng and Kessissoglou (2004), studied the free vibration analysis of a cracked beam by FEM. The mode shapes and natural frequencies of a cracked beam are obtained using finite element method.

Various effects of cracks on dynamic characteristics of a composite beam with reinforced polyamide were analyzed by Kisa (2004). Implementation of FEM and component mode synthesis method was made to this model.

Kisa et al. (2007) proposed a model related to the modal analysis of beams with circular cross section and containing non propagating open cracks which is a combination of FEM and component mode synthesis. In his work, beam is discretized into various parts starting from the crack sections and making a coupling by flexibility matrices taking the interaction forces derived from the fracture mechanics theory.

Influence of the double crack on the dynamic behavior of the cracked beam was studied by Yoon et al., (2007) and investigated both analytically and experimentally.

CHAPTER 3

THEORY AND FORMULATION

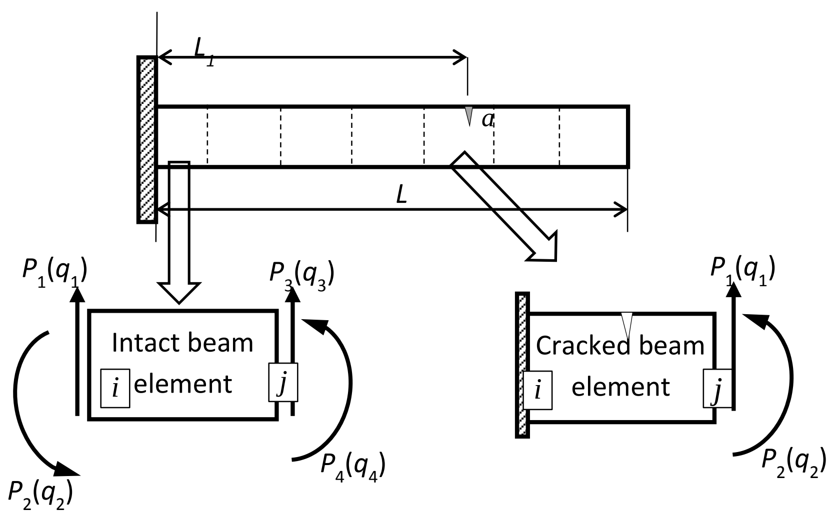
THEORY AND FORMULATION

3.1 Introduction

Cracks weaken the structures. As the crack dimension increases over the time, the structure becomes weaker than its original configuration due to flexibility. At the end, the structure may fail under service loads. Hence to have the knowledge of dynamic behavior of cracked members is very crucial for the safety and reliability of a structure.

3.2 Mathematical Formulation for Uniform Beam of Circular Cross-section:

A uniform cracked beam component of circular cross-section of diameter '*D*' and crack of depth '*a'* is taken for the calculation. The left hand side end node '*i'* is fixed and the other side node '*j'* is subjected to shear force P_1 and moment P_2.



q_1 = deflection at node i

q_2 = slope at node i

q_3 = deflection at node j

q_4 = slope at node j

L_e = length of the element

(FIG 1: Uniform beam is divided into elements and free body diagrams of intact and cracked beam element with 2 degrees of freedom per node)

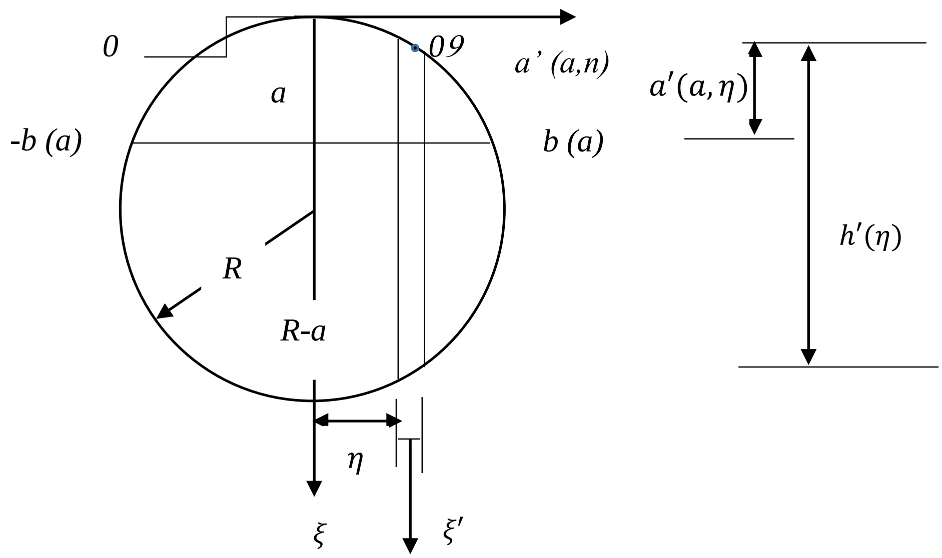


FIG 2: A cracked beam element subjected to shear force and bending moment (Cross-sectional view)

The mathematical statements of the vibration analysis of the uniform beam with an open transverse crack are analyzed based on the model proposed by Zheng D. Y. et al. (2004).The geometrical dimensions are:

$$\varphi' = \varphi + \sqrt{\frac{D2}{4}} - \eta 2 - \frac{D}{2}$$

$$\text{b (a)} = \sqrt{Da - a^2}$$

$$\text{h' } (\eta) = \sqrt{D^2 - 4\eta^2}$$

$$a' \ (a,\eta) = \sqrt{\frac{D^2}{4} - \eta^2} - \left(\frac{D}{2} - a\right)$$

Where D is the diameter of the beam. A similar approach suggested by Zheng D. Y. et al. (2004) to a circular cross-sectional beam for the derivation of the overall additional flexibility matrix as taken for the rectangular cross-sectional beam.

The additional strain energy due to the existence of the crack can be expressed as

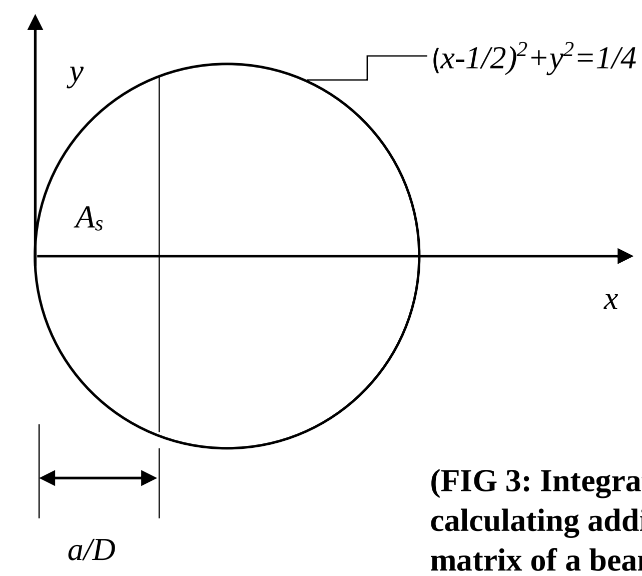
$$\Pi_c = \int G dA = \int_{-b(a)}^{b(a)} \left[\int_0^{a'(a,\eta)} G d\varphi' \right]$$

Where, 'G'-strain energy release rate function and 'A_c'effective cracked area. The strain energy release rate function G can be related to the stress intensity factors as mentioned above in which

$K_{I1}=\frac{4P1}{\pi D^2}\sqrt{\pi\varphi'}F_1\left(\frac{\varphi'}{h'}\right)$, $\quad K_{I2}=\frac{32}{\pi D^4}L_C P_2 h'\sqrt{\pi\varphi'}F_2\left(\frac{\varphi'}{h'}\right)$

$K_{I3}=\frac{32}{\pi D^4}P_3 h'\sqrt{\pi\xi'}F_2\left(\frac{\xi'}{h'}\right)$ $\quad K_{II2}=\frac{4}{\pi D^2}P_2\sqrt{\pi\xi'}F_{II}\left(\frac{\xi'}{h'}\right)$

Where ξ' is the penetrating depth of the strip.



(FIG 3: Integration area A_s for calculating additional flexibility matrix of a beam with circular cross-section)

The following expression can be derived as by Zheng D. Y. and Kessissoglou N. J. K. (2004)

$$C_{ij}=\frac{1}{E\prime}\frac{\angle^{2}}{\angle Pi\angle Pj}\int_{-\sqrt{Da-a^{2}}}^{\sqrt{Da-a^{2}}}\int_{0}^{\sqrt{\left(\frac{D^{2}}{4}-\eta^{2}\right)-(\frac{D}{2}-a)}}\left\{\left[\frac{4P1}{\pi D^{2}}\sqrt{\pi\xi'}F1\left(\frac{\xi'}{h'}\right)+\frac{32PcLch'}{\pi D^{4}}\sqrt{\pi\xi'}F2\left(\frac{\xi'}{h'}\right)+\frac{32P3h'}{\pi D^{4}}\sqrt{\pi\xi'}F2(\frac{\xi'}{h'})\right]^{2}+\frac{16P2^{2}}{\pi D^{4}}\xi'FII^{2}(\frac{\xi'}{h'})\right\}d\xi'd\eta\text{ , (i,j= 1,2,3)}$$

Where A_s is the integration area in a unit circle as shown in the figure. The suitable least square best fitted formulas are:

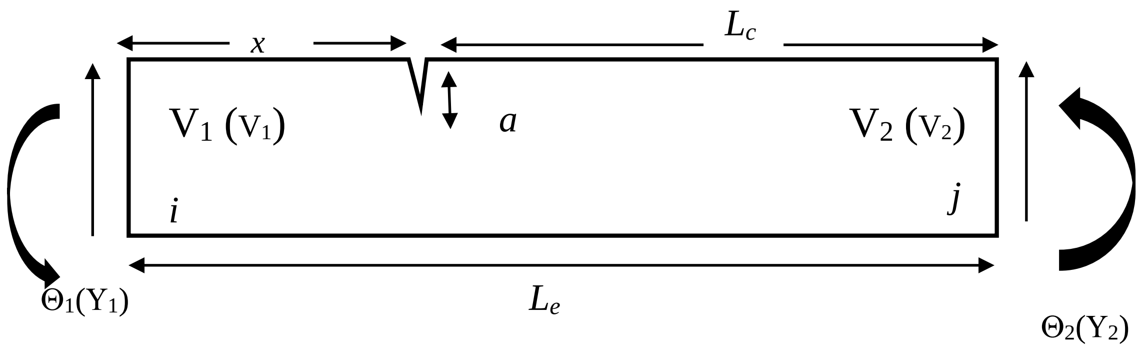
$$F(1,1)\approx e^{1/(1-x)}(-0.12323x^{0.4}+3.156480x^{0.8}+34.490509x^{1.2}+211.429280x^{1.6}-802.944428x^{2}+1964.215885x^{2.8}+3036.531592x^{3.2}-1692.594137x^{3.6}+411.505609x^{4})$$

$(0\leq x=a/D\leq 0.5)$

$$F(1,2)\approx e^{1/(1-x)}(0.0525646\,x^{0.4}-1.694740\,x^{0.8}+22.910177x^{1.2}-171.535649x^{1.6}+789.046673x^{2}-2318.500920\,x^{2.4}+4461.869140x^{2.8}-5337.583060x^{3.2}+3599.915932\,x^{3.6}-1044.227437x^{4})\ (0\leq x=a/D\leq 0.5)=F(1,3)$$

$$F(2,2)\approx e^{1/(1-x)}(-0.0181106x^{0.4}+0.483199x^{0.8}-5.519102x^{1.2}+35.485789x^{1.6}-141.871055x^{2}-367.85339x^{2.4}+610.901666x^{2.8}+639.711620x^{3.2}-384.398763x^{3.6}-98.728659\,x^{4})+\left(Lc/D\right)^{2}F(3,3)\quad(0\leq x=\mathrm{a/D}\leq 0.5)$$

$$F(3,3)\approx e^{1/(1-x)}(-0.102895x^{0.4}+3.653566x^{0.8}-53.161890x^{1.2}+423.977411x^{1.6}-2072.129084x^{2}-6447.218742x^{2.4}+13613.390334x^{2.8}-17873.887075x^{3.2}+12985.643127x^{3.6}-3999.17110x^{4})\ \left(0\leq x=\frac{\mathrm{a}}{\mathrm{D}}\leq 0.5\right)=F(2,3)$$



(FIG 4: A cracked beam element subjected to shear force and bending moment)

Flexibility matrix C_{intact} *of* the intact beam element subjected to shear force and bending Moment can be found out by

$$C_{intact} = \begin{bmatrix} \frac{L_e^{\ 3}}{3EI} & \frac{L_e^2}{2EI} \\ \frac{L_e^2}{2EI} & \frac{L_e}{EI} \end{bmatrix}$$

Total flexibility matrix C_{tot} of the cracked beam element is shown as

$$C_{total} = C_{intact} + C_{ovl}$$

$$C_{total} = \begin{bmatrix} \frac{L_e^3}{3EI} + C_{11} & \frac{L_e^{\ 2}}{2EI} + C_{12} \\ \frac{L_e^2}{2EI} + C_{21} & \frac{L_e}{3EI} + C_{22} \end{bmatrix}$$

Stiffness matrix K_c of a cracked beam element

From the equilibrium conditions as shown in the figure

$$(V_1 \theta_1 V_2 \theta_2)^T = [L](V_2 \theta_2)^T$$

Where the transformation matrix is

$$L = \begin{bmatrix} -1 & 0 \\ -L_e & -1 \\ 1 & 0 \\ 0 & 1 \end{bmatrix}$$

Hence the stiffness matrix K_C of a cracked beam element can be obtained as

$$K_C = L\ C_{tot}\ L^{-1}$$

Where L is defined as the transformation matrix of the beam for equilibrium conditions. Mass and geometric matrix are found out by a similar procedure. It is assumed that the change in mass matrix and geometric matrix is negligible for cracks.

The equation of motion for an undamped free vibration analysis of a beam is

$$[M]u'' + [K]u = 0$$

The above equation can also be reduced to find the modal frequencies of the beam

$$[K_e] - \omega^2[M_e] = 0$$

Where M_e = mass matrix of cracked beam and

$$[M_e] = \begin{bmatrix} 156 & 22L & 54 & -13l \\ 22L & 4L^2 & 13L & -3L^2 \\ 54 & 13L & 156 & -22L \\ -13L & -3L^2 & -22L & 4L \end{bmatrix}$$

CHAPTER 4

EXPERIMENTAL ANALYSIS

EXPERIMENTAL ANALYSIS

4.1 Introduction:

Modal testing of beam with only one crack is done to favor the results in light of FEM. The natural frequencies of the cracked beam nearby modal shapes are deciphered to know the effects of different parameters.

4.2 Specimen details:

Aluminum beam specimens of circular cross section are prepared with diameter 18mm and length 225mm. Artificial single crack is introduced at different positions e.g. 0.15L, 0.3L, 0.45L and 0.6L with the help of a cutting tool. Modal frequencies are measured at the mentioned positions separately. The depth of cracks e.g. 2mm, 4mm, 6mm, 8mm are introduced at a mentioned crack position .Four specimens for each location and depth were prepared.

4.3 Equipment Required for Vibration Test:

The equipment utilized as a part of the vibration measuring test set up, made by Bruel & Kjaer, Denmark contains the accompanying.

- Modal hammer
- Accelerometer
- FFT Analyzer
- Display Unit

Modal Hammer

The modal hammer energizes the structure with a uniform force with a given range of frequency. Five tips are given with impact hammer offering effect to the work piece. For present experiment, modal hammer type B&K 2302-5 was used.

FIG 5: Modal Impact Hammer (B&K type 2302-5)

Accelerometer

Miniature Delta Tron accelerometers is specifically designed to bear the industrious rugged environment. Combined high sensitivity, low mass and little physical structure make the accelerometer suitable for modal measurements, such as in aerospace, aviation and automobiles. Mounting clips are used to fit it in different structures. For the present analysis accelerometer (B&K 4507) was fixed on the free end of the specimen.

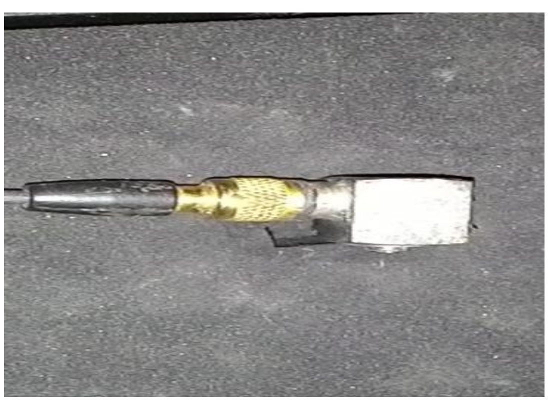


FIG 6: Accelerometer (B&K 4507)

Portable FFT analyzer (Type 3560B)

For the measurement of frequency for any structure four channels Bruel&Kjaer pulse analyzer system of type 3560 B was used. It is used specifically for the free vibration analysis of the specimen It has four channels to unite the links for the investigation of both data and yield signals. It is associated with the presentation unit.

FIG 7: FFT Analyzer (B&K3560 B)

Display unit

PC (Laptop) is mainly used as display unit. When the specimen was given impact in specific points by Impact hammer (B&K 2302-5), the signal of resulting vibrations were received by the accelerometer (B&K 4507) and they are directed to the FFT Analyzer. The output results from the analyzer were shown on the display unit graphically which incorporates graph of force amplitude spectrum, response amplitude spectrum, and coherence and frequency response functions.

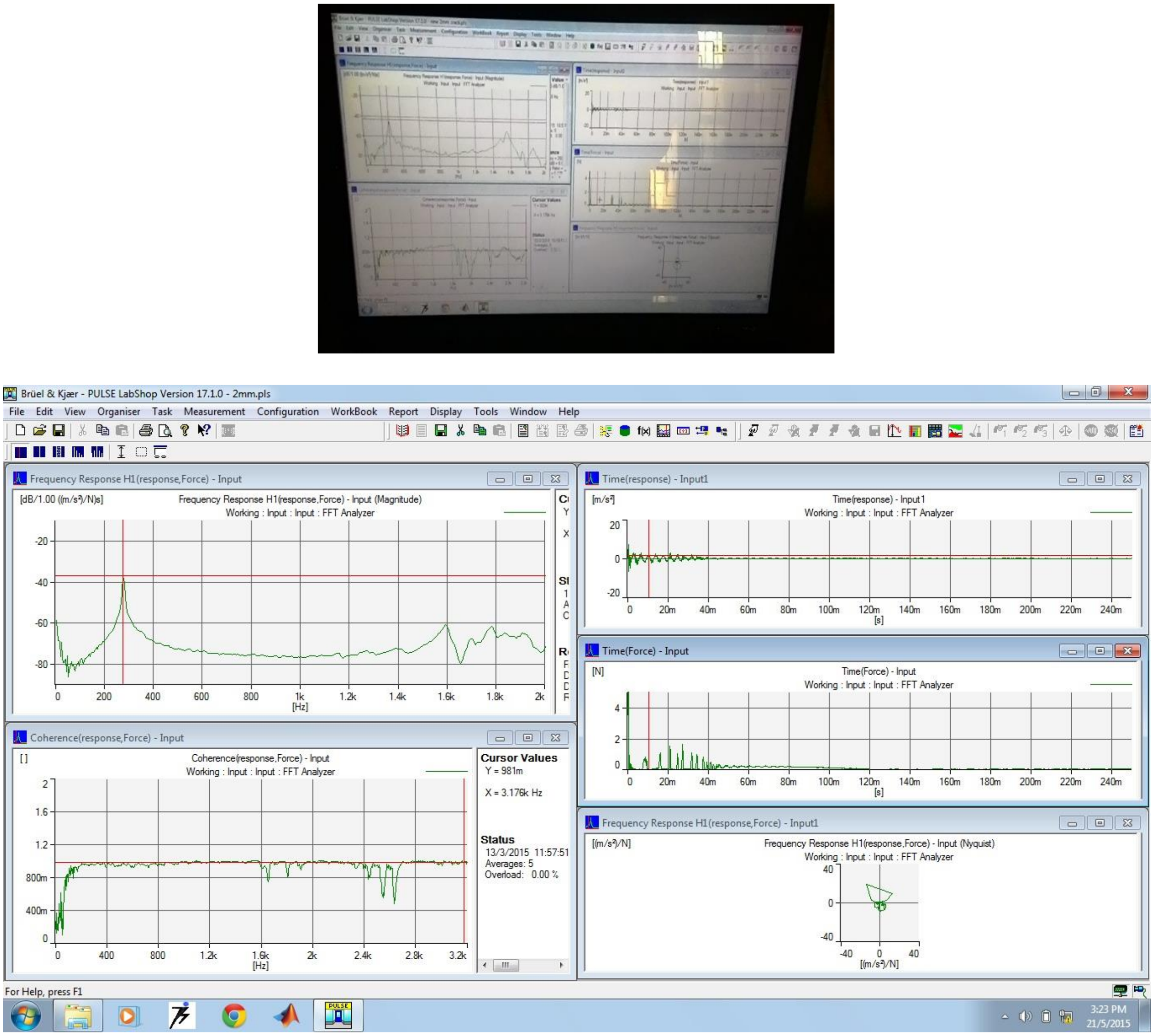


FIG 8: Display Unit

4.4 Setup and test procedure for vibration test

The cantilever end connection is used for the vibrational analysis. The specimen is fitted to the vice. The connections of FFT analyzer, laptop, transducers, modal hammer and cables to the system were made. The pulse lab shop version 10.0 software driver was inserted to the usbport of laptop. The accelerometer was placed on the beam at a point close to the free end of the specimen. The force was applied gently by using an impact hammer away from the accelerometer. The resulting vibrations of the specimens were measured by the accelerometer. Both data and yield signs are gathered by FFT Analyzer and output Frequency Response Functions (FRF) are transmitted to a PC for modal parameter extraction. The yield from the analyzer was shown on the display unit by the help of pulse software. Various forms of FRFs are directly measured. The peaks give the FRF average of five numbers of loading.The accuracy of the peaks obtained by FRF is checked by coherence. When the coherence is straight and close to one it indicates better FRF and if coherence is not straight the corresponding FRF is not accurate.

The vibration test setup is shown in the figure.

FIG 9: Vibration test setup, beam attached to the vice like cantilever beam, accelerometer and modal hammer in position

CHAPTER 5

RESULTS AND DISCUSSION

RESULTS AND DISCUSSION

5.1 Introduction:

Free vibration analysis (modal analysis) of cantilever euler-bernoulli beam with circular cross section is done. The comparative analysis with various crack depth and positions is done. Accuracy of this study is validated on the basis of a comparative analysis of the natural frequencies of the beams with the previous research papers. FEM and experimental analysis is made taking beam material as Aluminium (cantilever beam) with a transverse open crack in order to acquire the modes.

Results found from Ansys 15 and MATLAB environment have been discussed and investigated. Experiment is performed by FFT analyser.

The result contains

- Comparison with previous studies
- Results in both experimental and FEM analysis.

5.2 Comparison with previous studies

For the validation purpose of the present work and to analyze the results of the free vibration of the Bernoulli-Euler beam, the effects of various affecting parameters with a single crack are presented. A comparative analysis is made below with a previous research paper.

Free Vibration Analysis of Cracked Uniform Cantilever Beam

Young's modulus of material = 210GPa**,** Poisson's Ratio = 0.30**,** Material density = 7800 kg/m3**,** Width = 0.02m**,** Depth = 0.02 m**,** Length = 0.8m, Position of the crack from clamped end x_1= 0.12m, Depth of crack, a_1=0.002 m

Table 1: Comparison of Modal Frequencies for a cantilever beam with single crack

Modes	Natural Frequency (HZ) Shiffrin	Present analysis using FEM	% Error
Mode 1	26.1231	25.497	2.4
Mode 2	164.0921	159.52	2.78
Mode 3	459.6028	444.89	3.2

Table 2: Natural frequencies (1st mode) for cantilever Aluminum beam with or without cracks (L = 225 mm, D= *18mm*, E = 69 GPa, ρ = 2700 kg/m^3 and ν = 0.35)

Depth of Crack (in mm)	1st fundamental natural frequencies (ω_c/ω_i) at different position of cracks (L_1/L)							
	0.15		0.3		0.45		0.6	
	Ansys	FFT	Ansys	FFT	Ansys	FFT	Ansys	FFT
0mm	1	1	1	1	1	1	1	1
2mm	0.9945	0.9857	0.9968	0.971	0.9988	0.9857	0.9999	1
4mm	0.9710	0.9571	0.9841	0.9436	0.9931	0.9857	0.9982	0.9857
6mm	0.9284	0.9142	0.9578	0.9014	0.9807	0.9714	0.9945	0.9857
8mm	0.8575	0.8571	0.9128	0.8591	0.9576	0.9714	0.9862	0.9714

Table 3: Natural frequencies (2nd mode) for cantilever Aluminum beam with or without cracks (*L* = 225 mm, *D*= *18mm*, *E* = 69 GPa, *ρ* = 2700 kg/m³ and *ν* = 0.35)

Depth of Crack (in mm)	2nd fundamental natural frequencies (ω_c/ω_i) at different position of cracks (L_1/L)							
	0.15		0.3		0.45		0.6	
	Ansys	FFT	Ansys	FFT	Ansys	FFT	Ansys	FFT
0	1	1	1	1	1	1	1	1
2	0.9998	0.9974	0.9995	0.9975	0.9962	0.9924	0.9960	0.9951
4	0.9970	0.9949	0.9962	0.9951	0.9810	0.9899	0.9796	0.9879
6	0.9917	0.9924	0.9889	0.9951	0.9510	0.9849	0.9481	0.9710
8	0.9914	0.9874	0.9771	0.9902	0.9015	0.9798	0.8925	0.9228

FIG 10: Graphical presentation of 1st fundamental modal frequencies versus depth of crack in Ansys 15

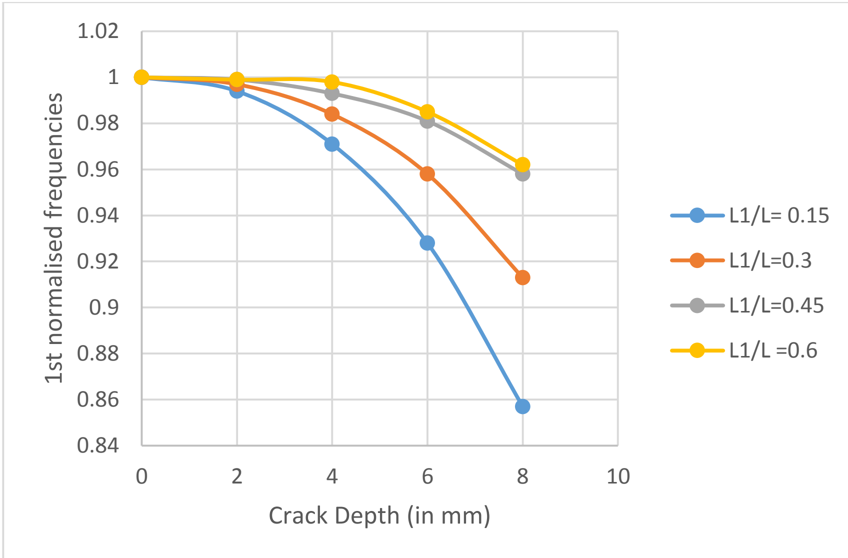


FIG 11: Graphical presentation of 1st fundamental modal frequencies versus depth of crack in FFT analyzer

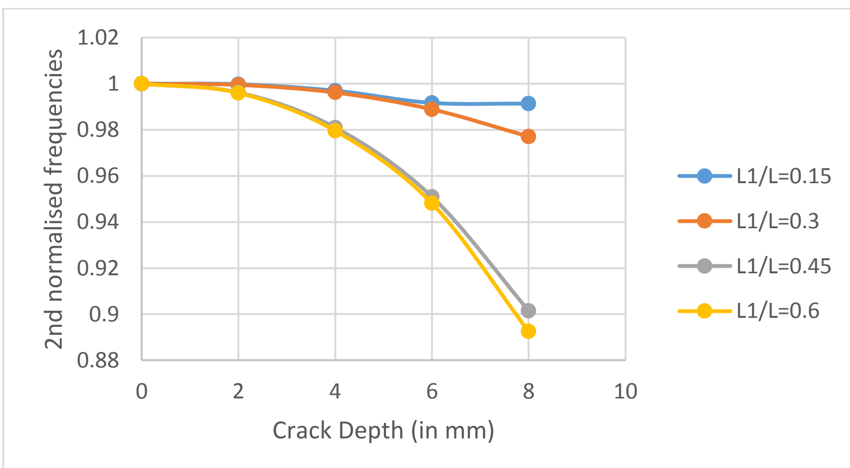
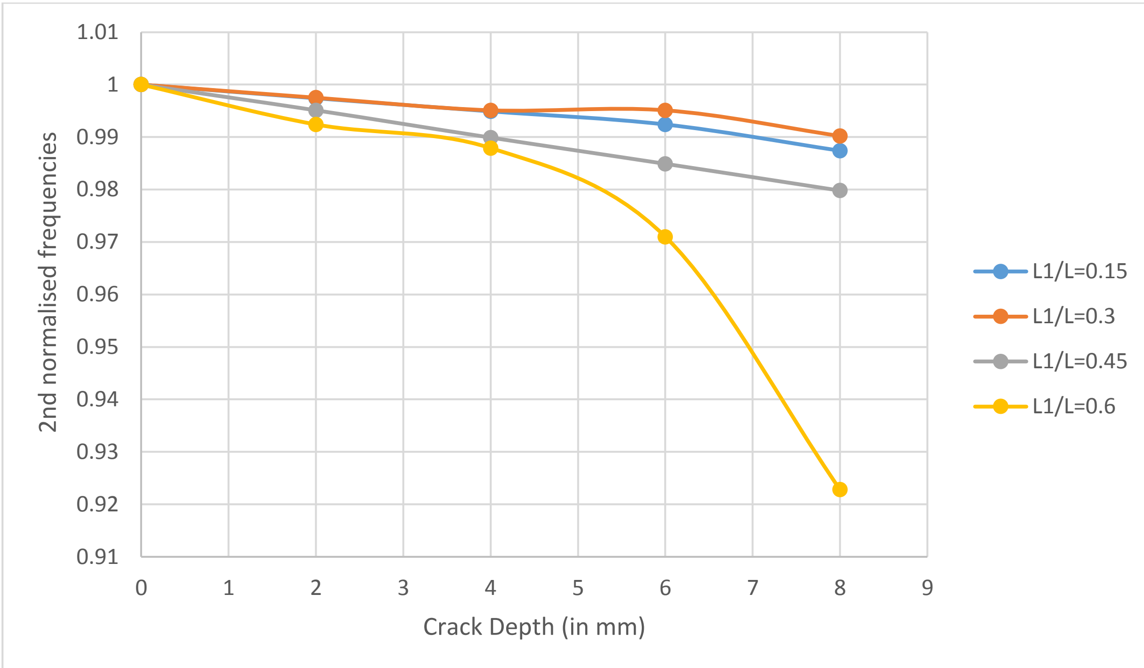


FIG 12: Graphical presentation of 2nd fundamental modal frequencies versus depth of crack in FFT analyzer



It is observed that for the 1st mode, the normalized mode frequency is more at the crack position closer to the free end. It decreases as for the crack position close to the fixed end. Frequency decreases as the depth of crack increases. The rate of decrement is more for the crack position closer to the fixed end. The rate of decrement decreases as the crack position moves away from the fixed position and it is minimum for the crack position close to the free end.

Similarly for the 2nd mode, normalized frequency is more at the crack position closer to the fixed end. It decreases for the crack position close to the free end. Frequency decreases as the depth of crack increases. The frequency decrement is more for the crack position closer to the free end. The rate of decrement decreases as the crack position moves away from the free end and it is minimum for the crack position close to the fixed end.

FIG 13: Graphical presentation of 1st fundamental modal frequencies versus position of crack in Ansys 15

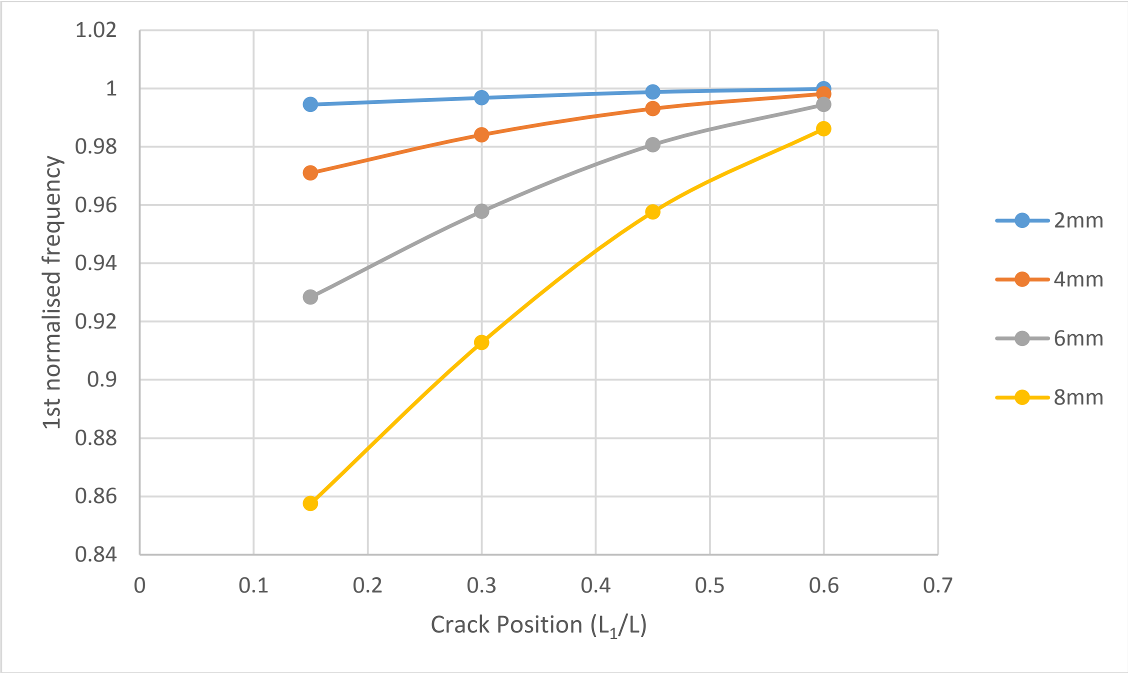


FIG 14: Graphical presentation of 1st fundamental modal frequencies versus position of crack in FFT analyzer

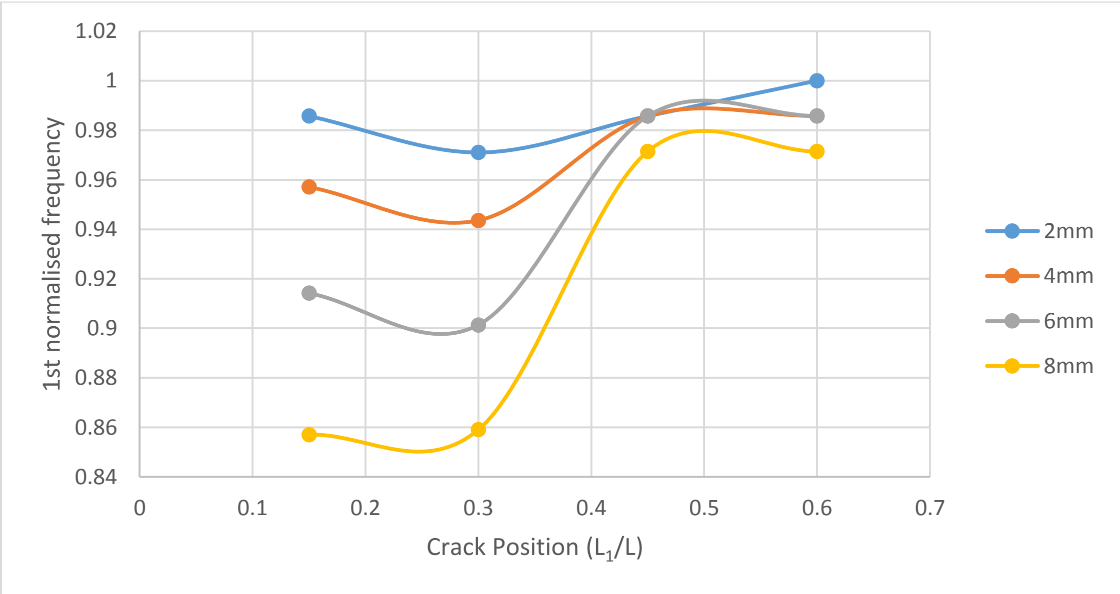


FIG 15: Graphical presentation of 2nd fundamental modal frequencies versus position of crack in Ansys 15

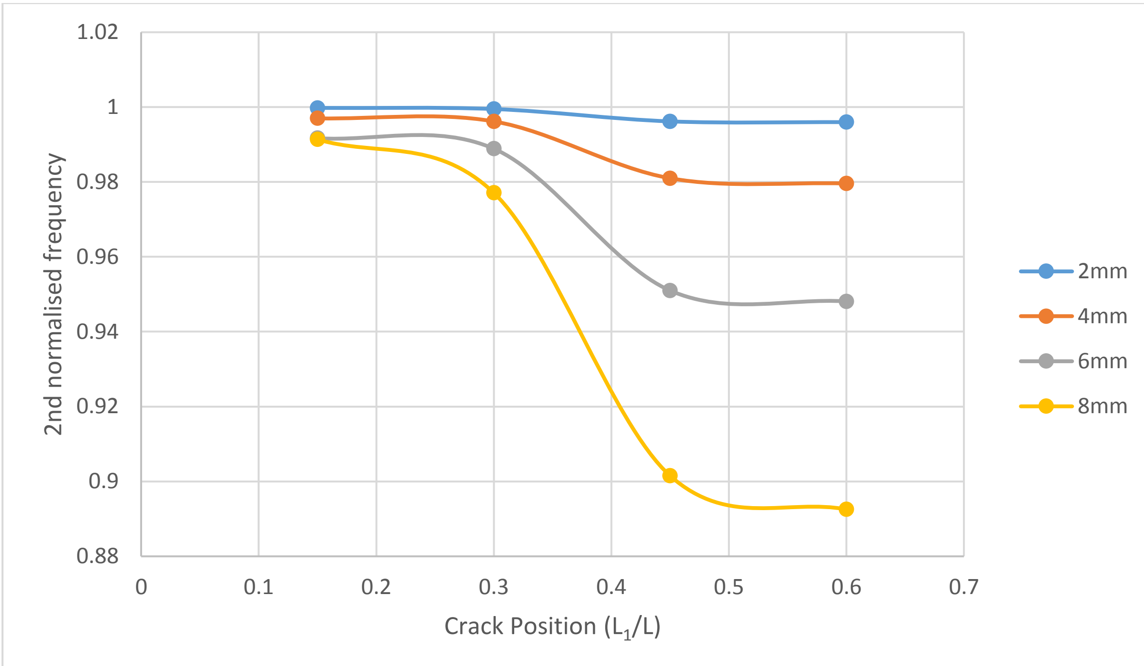
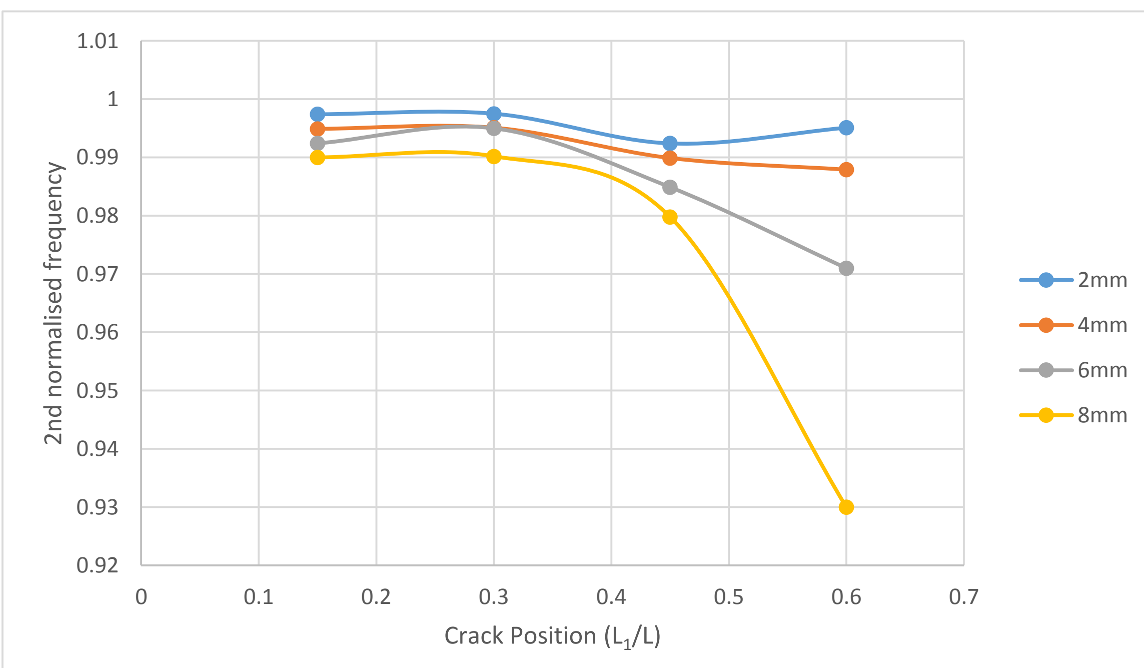
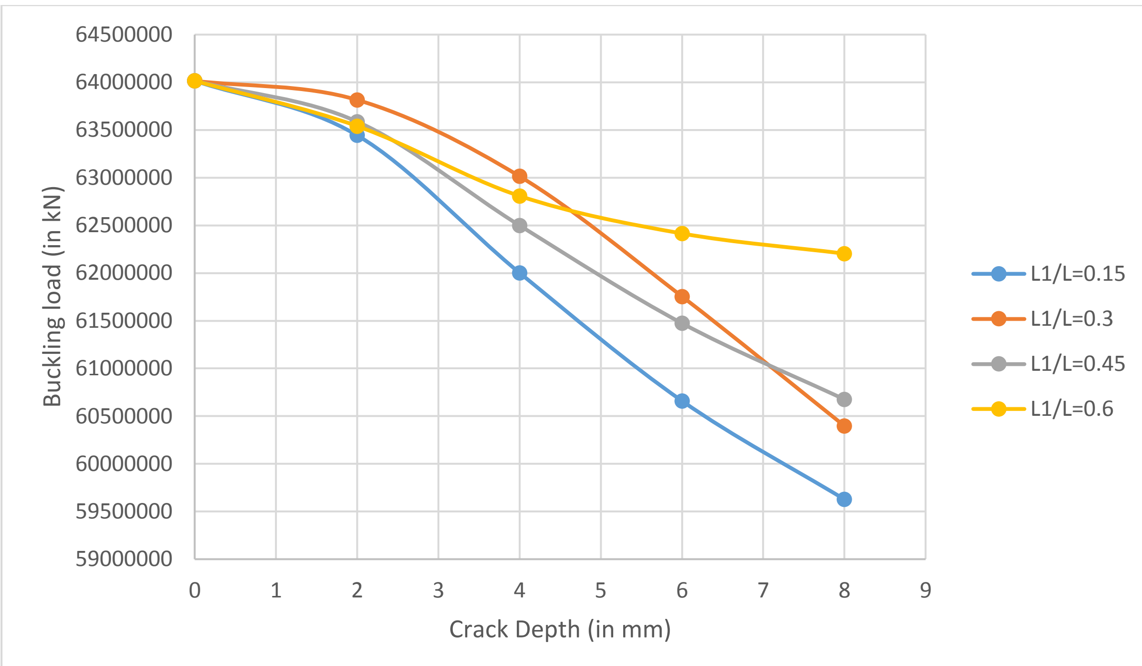


FIG 16: Graphical presentation of 2nd fundamental modal frequencies versus position of crack in FFT analyser



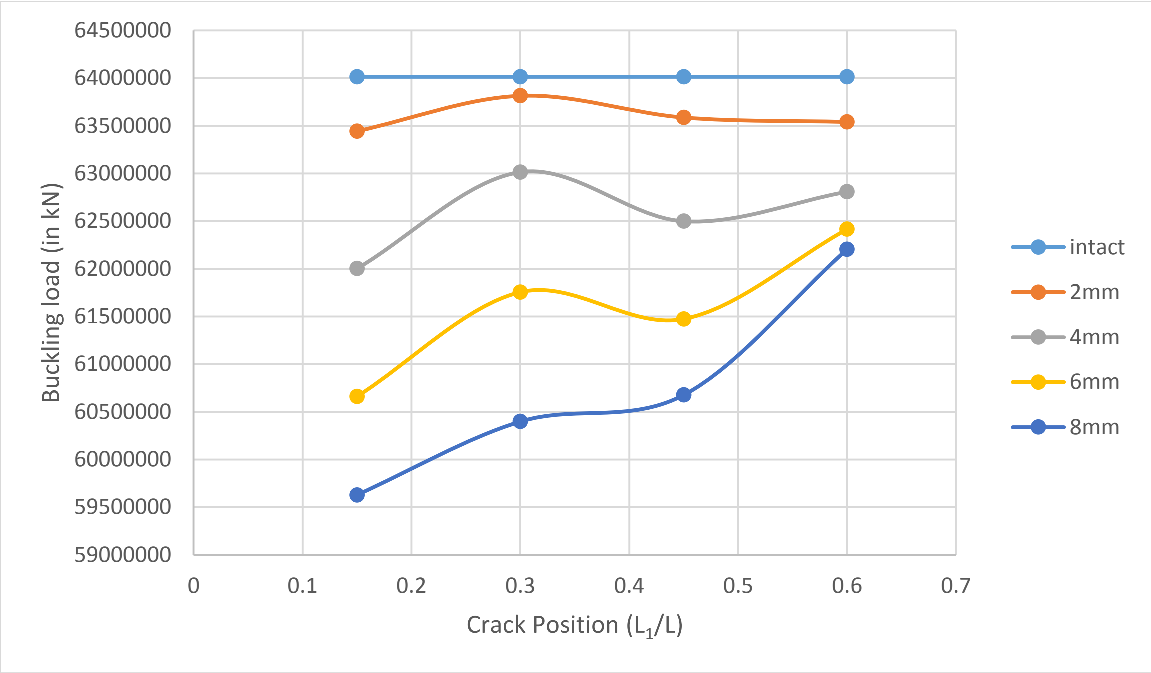
It is observed that for the 1st mode of normalized frequency the decrement in frequency is more for the crack position close to the fixed end where as for the 2nd mode the decrement is more for the crack position close to the free end. The decrement is minimum for the crack position close to the free end in case of 1st mode, whereas for the 2nd mode the decrement is minimum for the crack position close to the fixed end.

FIG 17: Graphical presentation of buckling load (P_{cr}) versus the depth of crack



It is observed that buckling load is more for the intact beam. The formation of crack decreases the buckling load of the beam. The more the depth of crack, the lesser the buckling load.

FIG 18: Graphical presentation of buckling load (P_{cr}) versus position of crack



It is observed that buckling load decreases as the depth of crack increases. The decrement in buckling load is more for the crack position close to the fixed end. The decrement of buckling load decreases as the crack position alters and moves away from fixed end to the free end.

CHAPTER 6

CONCLUSION

CONCLUSION

It is evident that frequency is maximum at the free end and frequency increases as the distance of crack from the fixed end increases. The effect of crack depth is maximum at the fixed end. As the distance of crack away from the fixed end increases the effect of crack depth decreases. At L1/L = 0.15, the frequency decreases by 1.35% for 2mm crack to 4mm crack. Similarly at L1/L=0.3, the frequency decreases by 1.27% for the same.

For the 2nd modal frequencies the result is quite different. The effect of crack depth on frequency increases as the distance increases from the fixed end. At $L_1/L = 0.15$, the frequency decreases by 0.27% for 2mm crack to 4mm crack. Similarly at $L_1/L=0.3$, the frequency decreases by 0.33% for the same.So it is notable that for the 1st modal frequencies effect of crack depth decreases but for the 2nd modal frequencies it's just the reverse.

Buckling load is maximum for the intact beam. As the distance of crack from the fixed end increases, the buckling load decreases. As the depth of crack increases, the buckling load decreases. The rate of decrement of buckling load is more at the fixed end and it decreases as the distance of crack from the fixed end increases.

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